On a : $|H_R^1| \leq \int_0^\pi \frac{e^{-2aR\sin(\theta)}}{R}\, d\theta \leq -\frac{2}{R}\frac{\pi}{4aR}\left[e^{-\frac{4aR}{\pi}\theta}\right]_0^\pi \xrightarrow[R\to\infty]{} 0$

$H_R^3 = -i\int_0^\pi Re^{-i\theta} e^{2ia\frac{1}{R}e^{i\theta}}\, d\theta$

On remarque que : $I_a = \int_0^\infty \frac{\cos 2ax}{x^2}\,dx$

$$\overset{(IPP)}{=} \left[-\frac{1}{2}\frac{1}{x}\left(-2a\sin(2ax)\right)\right]_0^\infty - \int_0^\infty \frac{a}{x}\sin(2ax)\,dx$$

On a : $\lim_{x\to 0} \frac{a}{x}\sin(2ax) = \lim_{x\to 0}\left(\frac{a}{x}\left(2ax + o(x^2)\right)\right) = 2a^2$

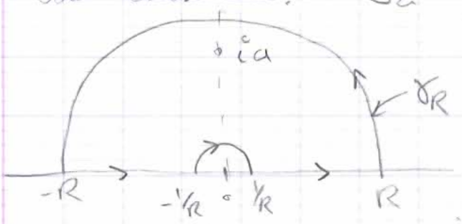
$$I_a = -2a^2 - \int_0^\infty \frac{a}{x}\sin(2ax)\,dx \underset{\hat{x}=ax}{=} -2a^2 - \int_0^\infty \frac{a^2}{x}\sin(x)\,a\,dx$$

$$= -2a^2 - a^3\int_0^\infty \frac{\sin(x)}{x}\,dx$$

$$= -2a^2 - a^3\frac{\pi}{2}$$

(6.2.4) $I_a = \int_0^\infty \frac{x^2-a^2}{x^2+a^2}\frac{\sin(x)}{x}\,dx = \frac{1}{2}\int_{-\infty}^\infty \frac{x^2-a^2}{x^2+a^2}\frac{\sin(x)}{x}\,dx$

On considère $J_a^R = \int_{\gamma_R} \frac{z^2-a^2}{z^2+a^2}\frac{e^{iz}}{z}\,dz$ $\boxed{R > a}$



$H_R^1 = \int_0^\pi \frac{R^2e^{2i\theta}-a^2}{R^2e^{2i\theta}+a^2}\frac{e^{iRe^{i\theta}}}{Re^{i\theta}}\, iRe^{i\theta}\, d\theta$

$H_R^2 = \int_{-R}^{-1/R} \frac{x^2-a^2}{x^2+a^2}\frac{e^{ix}}{x}\,dx$, $H_R^4 = \int_{1/R}^{R} \frac{x^2-a^2}{x^2+a^2}\frac{e^{ix}}{x}\,dx$

$H_R^3 = \int_\pi^0 \frac{\frac{1}{R^2}e^{2i\theta}-a^2}{\frac{1}{R^2}e^{2i\theta}+a^2}\frac{e^{i\frac{1}{R}e^{i\theta}}}{\frac{1}{R}e^{i\theta}}\, ie^{i\theta}\frac{1}{R}\, d\theta$